



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

May/June 2024

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 Find the roots of the equation $z^3 = -108\sqrt{3} + 108i$, giving your answers in the form $r(\cos \theta + i \sin \theta)$, where $r > 0$ and $0 < \theta < 2\pi$. [5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

2 Find the Maclaurin's series for $e^{1+x^2} + e^{1-x}$ up to and including the term in x^2 . [4]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps where letters can be written. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

3 It is given that

$$x = \sin^{-1}t \quad \text{and} \quad y = t \cos^{-1}t, \quad \text{for } 0 \leq t < 1.$$

(a) Show that $\frac{dy}{dx} = -t + \sqrt{1-t^2} \cos^{-1} t$. [3]

[illegible]

(b) Find $\frac{d^2y}{dx^2}$ in terms of t .

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

4 It is given that, for $n \geq 0$, $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \, dx$.

(a) Show that, for $n \geq 2$,

$$(n-1)I_n = \left(\frac{3}{5}\right)^{n-2} \left(\frac{4}{5}\right) + (n-2)I_{n-2}. \quad [5]$$

[You may use the result that $\frac{d}{dx}(\operatorname{sech} x) = -\tanh x \operatorname{sech} x$.]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.

(b) Find the value of I_4 .

[3]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 (2x - x^2) dx$. [4]

[illegible]

(c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

This image shows a full page of white paper with ten horizontal dashed lines, evenly spaced from top to bottom. The lines are thin and black, typical of primary school writing paper. There are no margins, text, or other markings on the page.

- 6 (a) Show that $(\cosh x + \sinh x)^{\frac{1}{2}} = e^{\frac{1}{2}x}$. [2]

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- (b) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5(\cosh x + \sinh x)^{\frac{1}{2}},$$

given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$. [10]

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- 7 (a) Use the substitution $u = 1 + x^2$ to find

$$\int \frac{x}{\sqrt{1+x^2}} dx. \quad [2]$$

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- (b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \sinh^{-1} x,$$

given that $y = 1$ when $x = 1$. Give your answer in the form $y = f(x)$. [10]

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- 8 (a) Find the set of values of a for which the system of equations

$$6x + ay = 3,$$

$$2x - y = 1,$$

$$x + 5y + 4z = 2$$

has a unique solution.

[2]

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- (b) Show that the system of equations in part (a) is consistent for all values of a .

[3]

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(d) Use the characteristic equation of $\mathbf{A}$ to show that

$$(14\mathbf{A} + 24\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A} + b\mathbf{I})^2,$$

where b is an integer to be determined.

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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